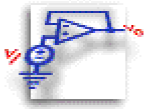

1 POWER DISTRIBUTION

Section 1.4a Least-Cost Power Transformer Sizing

Section 1.4.1 Introduction: Transformer Efficiency

Continued increases in power costs make the prediction of the total ownership cost of a transformer more important than ever. The total ownership cost includes the initial purchase price of the transformer and all costs associated with the operation of the transformer over its lifetime. Higher power costs increase the value of power dissipated as losses in transformers and other system equipment which increase the cost to own the device. High costs also cause consumers and suppliers of electric energy to examine if their equipment is operating at maximum efficiency. Even though transformers have efficiencies in the range of 97-99%, when compared to the other components in the power system, they account for a major percentage of the power losses. Since power transformers usually have long operational lives with little outage time, a small increase in the machine efficiency will cause significant economic benefit.

The losses in a power transformer are related to its loading. In the past, the recommended method of loading distribution transformers was to allow the load on the transformer to increase until the average copper temperature rose to 55°C. This is the self-cooled, oil-immersed rating (OA power rating) of the transformer. This loading level disregards the change in efficiency produced by different loading levels.

The maximum operating efficiency occurs at the load where the core losses equal the load losses. The core losses are related to the amount of iron in the transformer and are considered constant for a transformer on a power system operating at a constant voltage. The load losses are related to the amount of copper in the transformer windings and are a function of the load current. Increasing the transformer loading above this point decreases the efficiency and therefore increases the total cost to own and operate the device.

This section gives a method for computing the total cost to own a power transformer that includes all the important economic and technical factors.

Section 1.4.2 Transformer Loss Modeling

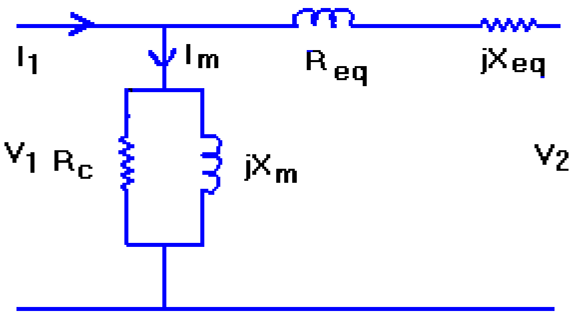

Fig. 1.4.1 Steady-state transformer model.

Figure 1.4.1 shows a per unit, steady-state transformer model. The model represents an approximate equivalent circuit for a single-phase transformer. It also represents the per phase equivalent for a three-phase transformer. Usually, resistance and reactance values are given in per unit (pu) on the power base of the transformer self-cooled (OA) power rating. **To review pu definitions, see Section 1.1.**

The series resistance, R_{eq}, represents the transformer power losses due to the resistance of both the primary and secondary windings. Multiplying the secondary winding resistance by a^2, where a is the turns ratio of the transformer, gives the equivalent value of the resistance of the secondary winding as seen from the primary side of the transformer. The value of R_{eq} is the sum of this calculation and the primary winding resistance.

The resistance of the windings can vary with the temperature of the windings. Heat build-up in the transformer core due to power losses causes the temperature rise. Reference [1] gives information on correcting the temperature effects. The value of R_{eq} represents the load losses of the transformer. The series inductive reactance, X_{eq}, models the effects of the leakage flux of both windings referred to the primary side. The shunt branches represent the core magnetizing effects. The shunt resistance, R_c, models the core losses due to eddy current and hysteresis effects in the core iron. The shunt inductive reactance, X_m, represents the magnetizing reactance required to produce the core flux.

The model provides the relationships between the primary-side voltage and the core losses and also the primary-side current and the load losses. The load losses are a function of the square of I_1, while the core losses are a function of the square of the primary voltage. For constant primary voltage, V_1, the core losses are constant. The transformer dissipates core losses even when no load is connected. If the power losses are in per unit, the load losses vary directly as the load varies, while the core losses vary inversely.

Equation (1.4.1) gives the transformer efficiency in percent from pu load and core losses.

where

$P_c := 0.02$ are pu transformer core losses, and

$P_l := 0.035$ are pu transformer load losses at the power output under consideration.

The values of P_c and P_l are typical values of transformer losses. Short-circuit and open-circuit tests provide the P_l and P_c data for transformers.

$$\eta_T := \frac{1.0}{1 + (P_c + P_l)} \cdot 100 \qquad \eta_T = 94.787 \qquad \textbf{(1.4.1)}$$

The core losses, P_c, are approximately constant for constant input voltage but the load losses, P_l, are a function of the transformer power output. Equation (1.4.1) gives the efficiency for a single level of loading. This is sufficient for acceptance testing, but predicting the efficiency under varying loads requires the load factor. The load factor is the ratio of average power output to rated power output. A general definition of load factor is given by Equation (1.4.2).

$$F_{LD} = \frac{P_{ave}}{P_{max}} \qquad \textbf{(1.4.2)}$$

where P_{ave} is the average load over a period, and P_{max} is the maximum load on the transformer over the same period. Examining the power metering information for a period of interest gives the information necessary to compute the load factor.

Equation (1.4.3) gives the transformer percent efficiency for any operating point given the per unit core and load losses **at rated load**.

Given a transformer with a load factor of

$$F_{LD} := 0.76$$

and pu full-load core and load losses of

$$P_c := 0.0135 \qquad P_l := 0.0418$$

This formula from [1] computes the efficiency as a function of load factor.

$$\eta_{TL}\left(F_{ld}, P_c, P_l\right) := \frac{1}{1 + \frac{P_c}{F_{ld}} + F_{ld} \cdot P_l} \cdot 100 \qquad \textbf{(1.4.3)}$$

Maximum Operating Efficiency

Plotting the result of Equation (1.4.3) for a number of load factors shows the pu load that gives the maximum transformer efficiency.

$$F_{LD} := 0.2, 0.25 .. 1.30$$

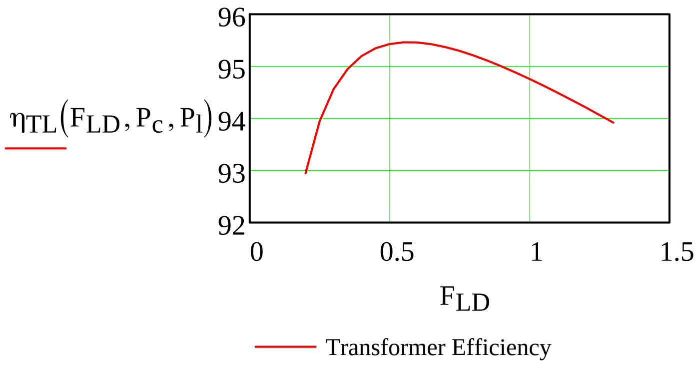

Fig. 1.4.2 Transformer efficiency as a function of load factor

Equation (1.4.4) from [1] gives the fraction of full load where maximum efficiency occurs.

Where L_{max} is the load factor that produces the maximum efficiency

$$L_{max}\left(P_c, P_l\right) := \sqrt{\frac{P_c}{P_l}} \qquad \textbf{(1.4.4)}$$

$$L_{max}\left(P_c, P_l\right) = 0.568$$

Loading a transformer to this point will produce the lowest amount of power losses and reduce the operating costs for the transformer.

If P_c and P_l are the pu losses at rated load, the value given by (1.4.4) is the pu load on the power base of the transformer that gives the most efficient operation. The ratio of P_l to P_c is called the loss ratio of a transformer.

Equation (1.4.5) from [1] gives the maximum efficiency for a given load factor. All variables are in pu.

$$\eta_{max}(LF, P_t) := \frac{LF}{LF + \left(\frac{P_t \cdot LF^2}{1 + LF^2}\right) \cdot 2} \cdot 100 \qquad \textbf{(1.4.5)}$$

where LF is the load factor in pu, and P_t is the total pu power loss at rated load. This value is a constant for a given transformer.

The loss ratio of a transformer determines at what load the maximum operating efficiency occurs. The plot of transformer efficiencies as a function of load factor for a number of different loss ratios produces a locus of maximum operating efficiencies for a transformer with constant total losses.

For a constant transformer loss of

$$P_T := 0.02 \text{ p.u.}$$

plot the efficiency curves of various loss ratios.

$$F_{ldmax} := 2$$

Define maximum load factor to plot.

$$j\_max := 20$$

Define the maximum number of points to plot.

$$j := 1 .. j\_max$$

Set the load factor increment

$$dl := \frac{F_{ldmax}}{j\_max}$$

and the load factor point array

$$F_{LD_j} := dl \cdot j$$

Compute the values of core losses, P_c, and load losses, P_l, for a constant total power loss, P_T.

$$dP := \frac{P_T}{j\_max}$$ loss increment

$$P_{c_j} := dP \cdot j \qquad P_{l_j} := P_T - dP \cdot j$$

Loss Ratio Plots for Constant Total Loss

Compute the loss ratios for the defined power increments.

$$LR_j := \frac{P_{l_j}}{P_{c_j}}$$

$n := 1 .. 10$

$$LR_n = \begin{pmatrix} 19 \\ 9 \\ 5.667 \\ 4 \\ 3 \\ 2.333 \\ 1.857 \\ 1.5 \\ 1.222 \\ 1 \end{pmatrix}$$

$m := 11 .. 20$

$$LR_m = \begin{pmatrix} 0.818 \\ 0.667 \\ 0.538 \\ 0.429 \\ 0.333 \\ 0.25 \\ 0.176 \\ 0.111 \\ 0.053 \\ 0 \end{pmatrix}$$

loss ratio array

These vectors are the results of the loss ratio calculations. Select the indices of the loss ratios 9, 1, and 0.25 to plot.

Efficiency curves for different loss ratios:

$$\eta9_j := \eta_{TL}\left(F_{LD_j}, P_{c_2}, P_{l_2}\right)$$ Loss Ratio = 9

$$\eta1_j := \eta_{TL}\left(F_{LD_j}, P_{c_{10}}, P_{l_{10}}\right)$$ Loss Ratio = 1

$$\eta025_j := \eta_{TL}\left(F_{LD_j}, P_{c_{16}}, P_{l_{16}}\right)$$ Loss Ratio = 0.25

Plot the maximum efficiency as a function of the load factor.

$$\eta mx_j := \eta_{max}\left(F_{LD_j}, P_T\right)$$

Plot the efficiencies for the various loss ratios on the same graph as the maximum efficiency curve.

$j := 1..19$

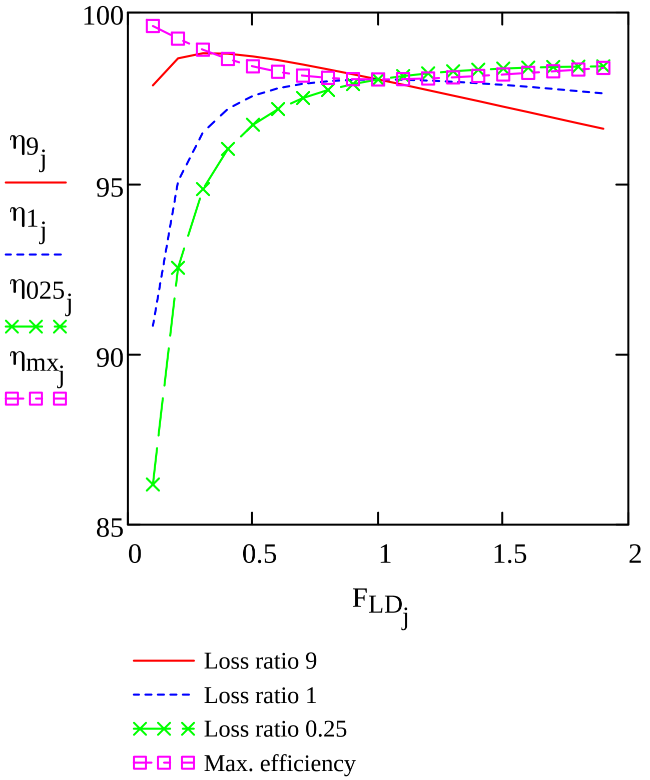

Fig. 1.4.3 Transformer efficiency and loss ratios

Figure 1.4.3 shows that high loss ratios produce maximum efficiency at low levels of transformer loading. High loss ratios imply that the $R_{eq} + jX_{eq}$ of Figure 1.4.1 are higher than the $R_{eq} + jX_{eq}$ of lower loss ratios. Low load factor transformers require higher $R_{eq} + jX_{eq}$. This higher series impedance also provides the transformer with more protection from external fault current damage by limiting the maximum available fault current in the circuit (To learn more about fault current calculations, see **Section 2.1**). The higher series impedance causes a higher percent regulation for the transformer. Regulation is given by Equation (1.4.6).

$$\text{Reg\%}(V_r, V_x, \theta) := V_r \cdot \cos(\theta) + V_x \cdot \sin(\theta) \ldots + \left[\frac{(V_x \cdot \cos(\theta) - V_r \cdot \sin(\theta))^2}{200} \right] \quad \textbf{(1.4.6)}$$

where V_r is the percent resistive voltage drop, V_x is the percent reactive voltage drop, and θ is the power factor angle of the load in degrees.

The plots in Figure 1.4.3 also indicate that the maximum efficiency for fully loaded transformers occurs only when the core losses and load losses are equal. This condition implies that a fully loaded transformer should have a loss ratio of 1 for maximum efficiency at full load.

Lower losses ratios mean less series impedance. Lower series impedance allows more external fault current which, in turn, means that breakers and other apparatus must have higher current interrupting ratings. The lower series impedance reduces the transformer regulation.

The load factor will be a guide to the most efficient transformer selection. In selecting the most economical transformer for a particular load, the load factor must be known. Figure 1.4.3 shows how the transformer efficiency varies over the selected range of load factors.